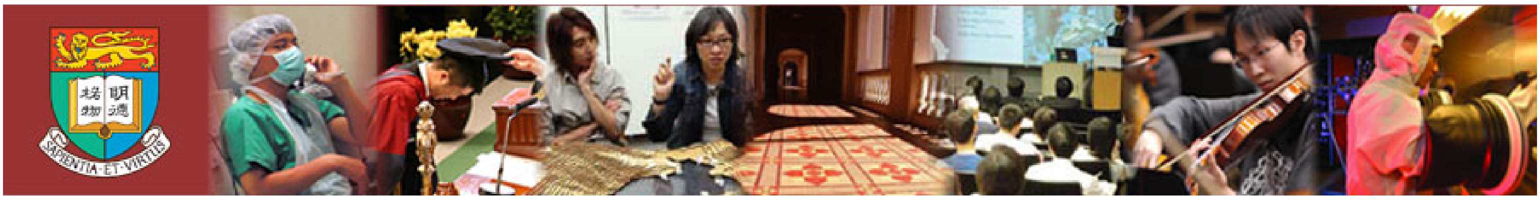

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Study of Displacement Current Effect for Planar Coils in Layered Medium

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***Abstract*—In this paper, a full-wave semi-analytical solution of self and mutual impedances for a coil-based system is proposed. It consists of two concentric circular coils with rectangular cross sections. Traditionally, the calculation of self and mutual impedances of these coils are based on eddy current approximation, where a simple background is assumed to achieve analytical and semi-analytical solutions. As the system operates at higher frequencies and with inhomogeneous media, the displacement current has to be taken into account. By deriving Maxwell's equations without eddy current assumption, the impedances of coils immersed in the layered medium can be obtained analytically. The results show a considerable discrepancy at sub-GHz frequencies, indicating the significant effect of displacement current at higher frequencies for a coil-based wireless power transfer link.**

I. INTRODUCTION

Analytical and semi-analytical solutions are always preferable in the mutual inductance calculation for coils with regular configurations, such as cylindrical symmetry and rectangular cross section. Previous research focused on coils with different shapes, coaxial and non-coaxial positions, as well as simple surroundings [1–4]. Since the derivation was based on the eddy current approximation, it is only suitable for low frequency application, for example battery charging.

Recently, higher frequency operation is of great interest as the density of electronic circuits grows and the exploitation of usable high frequency reaches terahertz (THz). The optimal frequency for wireless power transmission (WPT) into dispersive tissue was also reported to be above 1 GHz if the dimension of the transmit coil is much smaller than a wavelength [5]. In [5], the vertical magnetic dipole (VMD) model [6] was applied to describe a WPT system with two concentric loops. Note that VMD model is only suitable for a tiny current loop. For a current loop with relatively large size, a more general model can be constructed by solving the wave equation in cylindrical coordinates [7].

In this paper, we establish a full-wave analytical model to describe a two-coil system: the source and receiving coils are planar circular ones with rectangular cross sections, and they are immersed in layered medium. Thus, the influence of complex surroundings can be effectively modeled by fully considering the displacement current and the eddy current terms in Faraday's law. Based on this full-wave framework, the high frequency response of self and mutual impedances Z_{mn} can be captured.

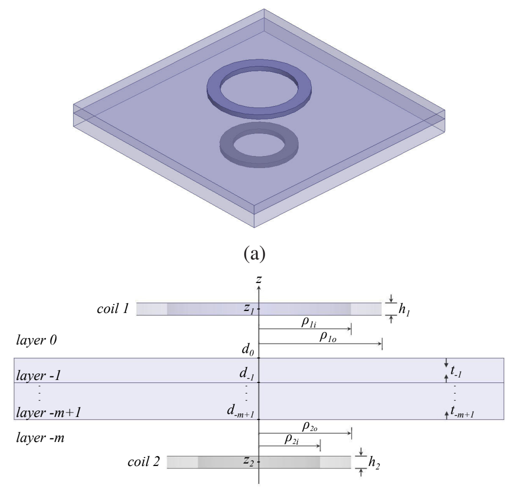

Fig. 1. Planar coils with rectangular cross section in layered medium. (a) 3 dimensional view. (b) Side view.

II. MODEL AND FORMULATION

Fig. 1 shows a WPT system with two circular and coaxial coils in layered medium. By assuming a time dependence $e^{-i\omega t}$ for all sources and fields, the electromagnetic fields due to a current source should satisfy

$$\nabla \times \mathbf{H} + (i\omega\varepsilon - \sigma)\,\mathbf{E} = \mathbf{J}, \tag{1}$$

$$\nabla \times \mathbf{E} = i\omega\mu\mathbf{H}. \tag{2}$$

Here, a system with two circular and coaxial loops is first considered. On the basis of cylindrical symmetry, only transverse electric (TE) modes are excited, and all the derivations below are conducted in the cylindrical coordinate system. By following the procedure in [7], $\mathbf{E}$ radiated by a current loop (transmitter) is given by

$$E_\varphi = \frac{-I\omega\mu\rho_1}{2}\int_0^\infty dk_\rho \frac{k_\rho J_1(k_\rho\rho_1)\, J_1(k_\rho\rho_2)\, B_m e^{-ik_{mz}z_2}}{k_{0z}}, \tag{3}$$

where detailed description of B_m is well documented in [6, 7]. According to the electromotive force introduced at the other loop (receiver), the mutual impedance between these two loops can be obtained as

TABLE I
DIMENSIONS FOR THE COILS (mm)

ρ_{1i}	ρ_{1o}	h_1	ρ_{2i}	ρ_{2o}	h_2	t_{-1}
10.0	11.0	0.035	20.0	21.0	0.035	10.0

$$-2\pi\rho E_\varphi = \mathcal{E} = ZI = (R + i\omega M)\, I. \tag{4}$$

Based on the mutual impedance between two loops and following the same procedure listed in [3], we can obtain the impedance formulation for the coil system as follows

$$Z = \frac{\omega\pi\mu}{h_1 h_2 \ln\left(\frac{\rho_{1o}}{\rho_{1i}}\right)\ln\left(\frac{\rho_{2o}}{\rho_{2i}}\right)} \int_0^\infty dk_\rho \left(\frac{k_\rho}{k_{0z}}\right) S\left(k_\rho\rho_{1o}, k_\rho\rho_{1i}\right) \times S\left(k_\rho\rho_{2o}, k_\rho\rho_{2i}\right) \int_{-\frac{h_1}{2}}^{\frac{h_1}{2}} d\tau_1 \int_{-\frac{h_2}{2}}^{\frac{h_2}{2}} d\tau_2 B_m e^{-ik_{mz}(z_2+\tau_2)}, \tag{5}$$

where $S\left(k_\rho\rho_2, k_\rho\rho_1\right) = \frac{J_0(k_\rho\rho_2) - J_0(k_\rho\rho_1)}{k_\rho}$. Note that the inner double integral can be solved analytically, after the positions of the coils are determined. Therefore, (5) is actually a single integral and can be evaluated numerically. The only difficulty in solving the impedances is the slow convergence of Sommefeld-type integrals (SIs) involved in (3) and (5). The simplest way to evaluate SIs is by deforming the Sommerfeld integration path (SIP) from the real axis into the complex k_ρ plane [6, 8], by virtue of Cauchy's theorem.

III. NUMERICAL DEMONSTRATION

For simplicity, but without loss of generality, we assume that the gap between the two coils is fully filled by a horizontally infinite dielectric plate characterized by ε_r and σ. This model can be also considered as a simplified WPT system for the biomedical implant application. The dimensions for these two coils are tabulated in Table I.

Fig. 2 show the calculated frequency response of mutual inductance between the two coils. When the frequency is below 10 MHz, all the impedances calculated by the proposed method with and without substrate have a good agreement with the reference (eddy current approximation), indicating a negligible displacement current in low frequency regime. However, they begin to oscillate as the frequency rises, implying the contribution of the displacement current becomes significant. For an air-backed case, the corrected solution of M_{21} shows a considerable discrepancy up to 10% at sub-GHz, in comparison with the solution of eddy current approximation. Moreover, for the cases with coils immersed in layered medium, M_{21} shows a speedy decrease at sub-GHz, due to the dielectric and radiation losses. Note that there exists a peak value of M_{21} ($M_{21} = 8.2929\ nH$ when $f = 1.5849 \times 10^8$ Hz), which is consistent with the conclusion in [5].

IV. CONCLUSION

A semi-analytical model for a planar two-coil system has been proposed to exploit the effect of displacement current at high frequencies. Based on the TE modes excited by a loop current source over layered medium, the impedance between two coaxial coils can be extracted through equivalent circuit theory. Numerical result verified that the effect of displacement currents at high frequencies is significant and cannot be ignored during the calculation, especially when the dielectric permittivity of human tissues is relatively large.

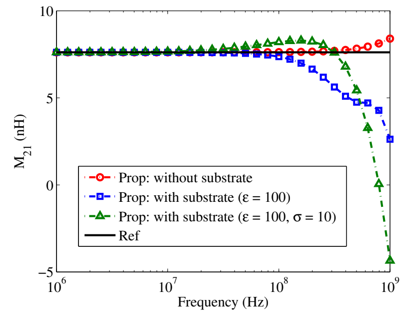

Fig. 2. M_{21} of the coil system: reference (Ref, displacement current excluded) solution and proposed (Prop, displacement current included) solutions.

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